COMMON PRE-BOARD EXAMINATION 2022-23

Subject: MATHEMATICS (041)

SET- 1

Maximum Marks:80 *Time allowed: 3hours*

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (**VSA**)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (**SA**)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (**LA**)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Q. No	SECTION A (MCQ)	Marks
1.	If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then the value of $\left|2\hat{a}+\hat{b}+\hat{c}\right| =$	1
	A 4 **B** 6 **C** 16 **D** 10	
2.	Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called ...	1
	A objective **B** Domain **C** optimal solution **D** constraint	
3.	The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5) (15, 15) and (0, 20). Let Z= px + qy, where p, q> 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is	1
	A $p=q$ **B** $p=2q$ **C** $q=2p$ **D** $q=3p$	
4.	Evaluate: $\int_{-1}^{1} \frac{1}{1+x^2}dx$	1
	A $\frac{\pi}{4}$ **B** 1 **C** 0 **D** $\frac{\pi}{2}$	

5. If $A = \begin{bmatrix} 3 & 8 \\ 2 & 5 \end{bmatrix}$ and $AB = I$, where I is an identity matrix,, then B= 1

A $\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix}$ B $\begin{bmatrix} 5 & 8 \\ 2 & 3 \end{bmatrix}$ C $\begin{bmatrix} -5 & 8 \\ 2 & -3 \end{bmatrix}$ D $\begin{bmatrix} 5 & -8 \\ -2 & 3 \end{bmatrix}$

6. The domain of the function $cos^{-1}(2x-1)$ is 1

A [-1, 1] B [0, 1] C (-1, 1) D $[0, \frac{1}{2}]$

7. The value of x if A is a skew symmetric matrix, where $A = \begin{pmatrix} 0 & 4 & -3 \\ 0 & 0 & 2 \\ x & 1 & 0 \end{pmatrix}$ is 1

A 3 B 2 C -3 D 4

8. If C_{ij} is the cofactor of a_{ij} for the matrix $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & -3 \\ 0 & 2 & 1 \end{pmatrix}$, then $C_{21} + C_{22}$= 1

A 1 B -7 C 5 D 4

9. If $|\vec{a}| = a$, then $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times k|^2 =$ 1

A 0 B a^2 C $2a^2$ D $3a^2$

10. If $\sqrt{x} + \sqrt{y} = 1$, then $\frac{dy}{dx}$ at $(\frac{1}{4}, \frac{1}{4})$ = 1

A 0 B 1 C $\frac{1}{2}$ D -1

11. Evaluate: $\int_0^1 xe^x \, dx$ 1

A 0 B 1 C e D 2

12. If three dice are thrown together, then the probability of getting at least one even number is equal to 1

A $\frac{7}{8}$ B $\frac{1}{216}$ C $\frac{1}{8}$ D $\frac{3}{8}$

13. The sum of degree and order of differential equation $\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = x^2$ is 1

A 3 B 4 C 5 D not defined

14. The angle between the lines $2x = 3y = -z \; and \; 6x = -y = -4z$ is — 1

A $\frac{\pi}{3}$ B $\frac{\pi}{4}$ C $\frac{\pi}{2}$ D $\frac{\pi}{6}$

15. If $f(x) = \begin{cases} \frac{k\cos x}{2\pi - x}, & x \neq \frac{\pi}{2} \\ 2, x = \frac{\pi}{2} \end{cases}$ is continuous at x= $\frac{\pi}{2}$ then value of k is — 1

A 2 B -2 C 4 D -4

16. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then value of x — 1

A ± 6 B $\pm 2\sqrt{6}$ C ± 3 D $\pm 6\sqrt{2}$

17. The matrix $A = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{pmatrix}$ is a ______matrix. — 1

A *scalar* B *unit* C *diagonal* D *square*

18. Integrating factor of the linear differential equation $x\frac{dy}{dx} - y = 2x$. — 1

A $-x$ B x C $logx$ D $\frac{1}{x}$

ASSERTION-REASON BASED QUESTIONS

In the following questions (19 and 20), a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

A) Both A and R are true and R is the correct explanation of A.

B) Both A and R are true but R is not the correct explanation of A.

C) A is true but R is false.

D) A is false but R is true.

19. (A) The points $A(2,3,-4), B(1,-2,3)$ and $C(3,8,-1)$ are collinear.
(R) If the points A, B and C are collinear, then direction ratios of AB and AC ae proportional — 1

A B C D

20. (A) If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors and $\hat{a} + \hat{b} + \hat{c} = 0$, then $\hat{a}.\hat{b} + \hat{b}.\hat{c} + \hat{a}.\hat{c} = -\frac{3}{2}$.
(R) For any two vectors, $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{b} = \vec{b}.\vec{a}$. — 1

A B C D

SECTION B

21. Show that $f(x) = cosx, f: R \rightarrow R$ is neither one to one nor onto. 2

OR

Simplify: $sin^{-1}\left(sin\frac{2\pi}{3}\right) + cos^{-1}\left(cos\frac{7\pi}{6}\right) + tan^{-1}(tan\frac{3\pi}{4})$

22. If $y = \log(x + \sqrt{1+x^2})$, then prove that $\sqrt{1+x^2}\frac{dy}{dx} = 1$ OR 2

If $x = e^{cos2t}, y = e^{sin2t}$, show that $\frac{dy}{dx} = -\frac{ylogx}{xlogy}$.

23. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long. 2

24. Find a unit vector which is perpendicular to both of the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ *and* $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$. 2

25. Write the cartesian equation and vector equation of a line which passes through the point (-2, 4, -5) and parallel to the line $\frac{3-x}{2} = \frac{y-4}{2} = \frac{z}{6} = \mu$. 2

SECTION C

26. Evaluate using properties of integrals: $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1} dx$ OR 3

Evaluate using properties of integrals: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{cosx}{1+e^x} dx$

27. Find the particular solution of the differential equation $\frac{dy}{dx} + ycotx = 4xcosecx$, given that $y = 0$ *when* $x = \frac{\pi}{2}$. 3

28. Using LPP maximise: z = 60x + 40y 3

Subject to: $x + 2y \leq 12, 2x + y \leq 12,\ x \geq 0, y \geq 0$.

29. A card from a pack of 52 cards is lost. From the remaining cards of the pack two cards are drawn and found to be both spades. Find the probability that the lost card being spade. 3

30. Evaluate: $\int \frac{x^2+1}{x^2-5x+6} dx$ OR Evaluate: $\int \frac{e^x(2+sin2x)}{1+cos2x} dx$ 3

31. Evaluate: $\int \frac{x}{1+xtanx} dx$ 3

SECTION D

32. Determine the relation R defined on set of real numbers R as $R = \{(a,b): a - b + \sqrt{2} \in S\}$, where S is the set of irrational numbers is reflexive, symmetric and transitive. Justify your answer. 5

OR

Let $\{1,2,3,4,5,6,7,8,9\}$ and R be the relation in $A \times A$ defined by $(a,b)R(c,d)$ if $a + d = b + c$, $(a,b),(c,d) \in A \times A$.
Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)].

33. Find the area of the region 5

$$\{(x,y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$$

34. Find the image of the point having position vector $\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ in the line $\vec{r} = \hat{\jmath} + 2\hat{k} + t(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$. 5

OR

The point on a line given in the form $(2\mu + 1, 1 - \mu, \mu)$.

i) Write the equation of the line in vector form?

ii) Is the point $\left(0, \frac{3}{2}, -\frac{1}{2}\right)$ lie on the given line? Why?

iii) Find the shortest distance between the given line and the line whose equation is $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \lambda(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$.

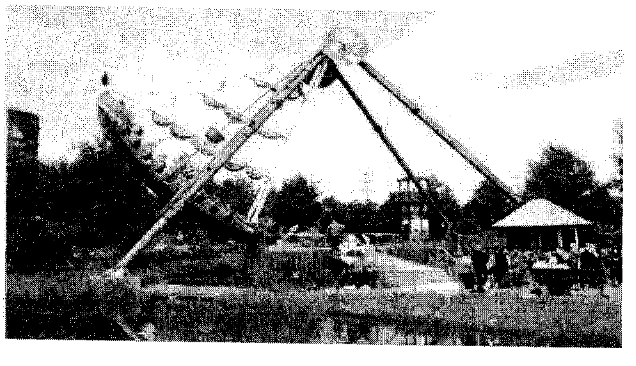
35 If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, then find AB and use this to solve the system of equations $x - y = 3, 2x + 3y + 4z = 17$ and $y + 2z = 7$. 5

SECTION E

This section comprises of with two sub-parts. First two case study questions have three sub-parts i, ii and iii of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. Ravi visited an exhibition along with his family. The exhibition had a huge swing. Ravi found that the swing traced the path of a Parabola as given by 4

$f(x) = ax^2 - 1, a > 0$



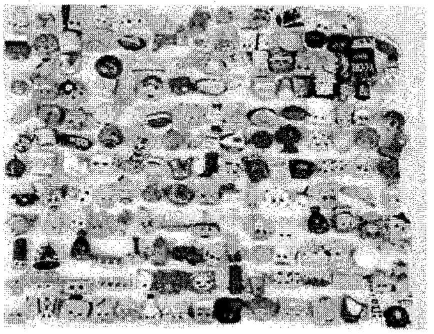
i. Find a if (2, 0) is a point on the parabola.

ii. What is the range of f(x).

iii. Find the intervals in which the function f(x) is strictly increasing or decreasing. OR

iii. Find the absolute maximum or absolute minimum in the interval [-1, 3]. Also find the absolute maximum and minimum values of the function.

37. A manufacturer making toys can sell x items at a price of ₹ $\left(5-\frac{x}{500}\right)$ each. 4



Cost price of one item is ₹ $\left(\frac{6}{5}+\frac{500}{x}\right)$.

Based on the above answer the following:

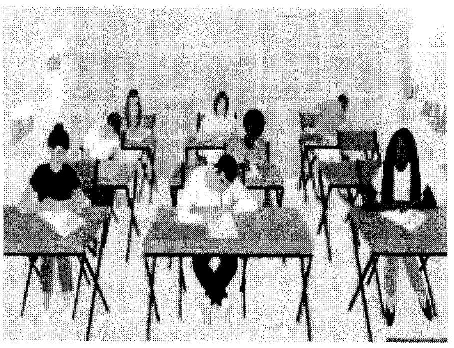
i. Write the revenue function s(x) and cost function c(x) in terms of x where x is the number of items sold.

ii. Find the profit obtained by selling 500 items.

iii. Find the number of items to be sold to get maximum profit. OR

iii. Find the maximum profit if the profit function p(x) = $-2x^2+92x-300$.

38. Three students A, B and C are trying to solve a problem independently. Probability of solving the problem independently by A, B and C are $\frac{4}{5},\frac{3}{4}$ and $\frac{2}{3}$ respectively. 4



Based on the above information answer the following:

Find the probability that

i. the problem is solved.

ii. exactly one of them solves the problem.

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